Indefinite Integration with "u" Substitution – The Chain Rule For Integration

$$\int f(g(x))g'(x)dx = \int f(u)\, du$$

"u" Substitution Procedure for Polynomial, Rational and Radical Functions

1.)   Let "u" be the value inside the quantity
2.)   Determine $\frac{du}{dx}$
3.)   Match $\frac{du}{dx}$ to the original function
4.)   Substitute
5.)   Find the antiderivative in terms of "u"
6.)   Re-substitute for "x"

General Power Rule with "u" Substitution

$$\int u^n\, du = \frac{u^{n+1}}{n+1} + c$$

Example 1: $\int (3x^2 + 9)^5 (6x)\, dx$

Example 2: $\int x^3 (x^4 + 3)^2\, dx$

Example 3: $\int t\sqrt{t^2 + 2}\, dt$

Example 4: $\int \frac{5x}{\sqrt{1-x^2}}\, dx$

Definite Integration with "u" Substitution

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Procedure for Definite Integrals with "u" Substitution

1.) Let "u" be the value inside the quantity
2.) Determine $\frac{du}{dx}$
3.) Match $\frac{du}{dx}$ to the original function
4.) Substitute
5.) Determine new upper and lower bounds in terms of "u"
5.) Find the antiderivative in terms of "u"
6.) Evaluate the integral – Do not resubstitute for "x"

Example 1: $\int_0^2 3x(3x^2 - 4)^3\,dx$

Example 2: $\int_{-1}^0 \frac{8x}{(2x^2+2)^2}\,dx$

Example 3: $\int_1^2 4x\sqrt{x^2 - 1}\,dx$

Example 4: $\int_0^4 \frac{x}{\sqrt{1+2x^2}}\,dx$

Difficult "u" Substitution Problems

Example 1: $\int x\sqrt{x+6}\,dx$

Example 2: $\int x^2\sqrt{1-x}\,dx$

Example 3: $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx$

Example 4: $\int \frac{2x+1}{\sqrt{x+4}}\,dx$

Example 5: $\int_1^2 (x-1)\sqrt{2-x}\,dx$

AP Style "u" Substitution Problems

Example1:   If f is continuous and $\int_0^4 f(u)\,du = 10$, find $\int_0^2 f(2x)\,dx$

Example2:   If f is continuous and $\int_0^9 f(u)\,du = 4$, find $\int_0^3 xf(x^2)\,dx$